Heat transport mechanisms for an end-heated aluminum rod

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The contributions from conduction, convection, and radiation for an end-heated aluminum rod were quantified with experimental considerations in mind. Multiple experiments were carried out to ascertain various physical properties of the system and the aluminum rod. By applying heat-flow theory, simulations, and data-fitting techniques, the specific heat capacity, conductivity, emissivity, convective heat transfer coefficient of the system, and thermal contact resistance between the power source and aluminum rod were determined.

Heat transport mechanisms describe the transfer of thermal energy between physical systems as a function of temperature and pressure. The fundamental modes of heat transfer are *conduction*, *convection*, and *radiation* which occur at the boundary between systems. For each of the fundamental modes, there is an associated physical property for an arbitrary material. In the application of materials in science and engineering, it is imperative that these physical properties are known.

The goal of this paper is to elucidate the contributions of the three fundamental modes of heat transfer within an end-heated aluminum rod system. Each contribution, *conduction*, *convection*, and *radiation*, is characterized by a physical property of the aluminum rod (heat transport parameters): κ, h_c, ϵ, respectively. Where, κ is the thermal conductivity, h_c is the convective coefficient of the system, and ϵ is the emissivity. The thermal contact resistance between the end heat-source, R_{th} and the specific heat capacity, c_{Al} is also discussed.

Section I delivers a basic mathematical introduction to heat flow theory and provides insight into how the heat transport mechanisms for an end-heated aluminum rod are coupled. Section II describes the experimental setup for all the experiments conducted and assumptions made for each one. Section III presents the results of the experimentation and provides the experimentally determined values for the heat transport parameters, c_{Al}, and R_{th} with experimental errors in mind. Section IV provides a conclusion with future considerations for further and more accurate experimentation.

I. THEORETICAL FRAMEWORK

The fundamental modes of heat transport in an aluminum rod can be characterized by a parabolic partial differential equation, the *heat equation*[1],

$$\frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial x^2} + \beta(u - u_{air}) + \gamma(u^4 - u_{air}^4) = 0. \tag{1}$$

Where $\alpha = \frac{\kappa}{\rho c_{Al}}$, ρ is the density of aluminum, $\beta = \frac{2h_c(L+r)}{\rho c_{Al} r}$, r is the radius of the aluminum rod, L is the total length of the rod, and $\gamma = \frac{2\epsilon\sigma(L+r)}{\rho c_{Al} r}$, σ is the Stefan-Boltzmann constant, which is assumed in this paper to be, $\sigma = 5.670373 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$. Equation 1 describes the distribution of heat (or variation in temperature) in a given region over time undergoing all three modes of heat transport. For this paper, $u = u(x, t)$ will be the temperature at point, x and time, t.

The solution to equation 1 (with the proper boundary conditions) fully describes both the transient phase of the effect of heat flow and temperature change and the static (equilibrium) phase of the same. However, analytically solving equation 1 is difficult. Thus, it is useful to isolate and develop a heat flow model each of the fundamental modes of heat transfer and consider them separately.

The following discussion of the three modes of heat transport were taken into consideration whilst designing the experiments (see Section II).

A. Conduction

Isolating conduction greatly reduces the complexity of equation 1, since the convection and radiation terms are removed,

$$\frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial x^2} = 0. \tag{2}$$

Equation 2 describes the time and spacial evolution of temperature in the aluminum rod subjected to certain boundary conditions. During experimentation, the end of the rod is heated with a constant power source which is delivering heat energy. Since the amount of heat energy per unit time, $\frac{dQ}{dt}$ is known, it is convenient to view conduction in the following form,

$$\frac{dQ}{dt} = -\kappa A_{\times} \frac{du}{dx}. \tag{3}$$

Where $A_{\times}$ is the cross sectional area. Equation 3 describes the amount of heat energy transported between two points on the rod. Note it is now clear to see that the unit of κ is $\mathrm{Wm^{-1}K^{-1}}$. The negative sign is indicative of the direction of heat transfer. For simulations, the discrete form of equation 3 is used (see Appendix A). The relationship between equation 2 and 3 is given by the specific heat equation,

$$\frac{dQ_{net}}{dt} = (c_{Al}\rho dV)u_{rise}. \quad (4)$$

Where $\frac{dQ_{net}}{dt}$ is the *net* heat transfer through an infinitesimal volume segment of the rod, and u_{rise} is the corresponding temperature increase of that point. So, using 3 and 4 one can derive 2.

Furthermore, there exists a finite contact thermal resistance between the heat-power source and the aluminum rod. Using the discrete form of equation 3, the absolute thermal resistance between multi-layered contact systems can be determined with,

$$\frac{\Delta Q}{\Delta t} = -\frac{(u_1 - u_2)}{\frac{\frac{\Delta x_1}{\kappa_1} + \frac{\Delta x_2}{\kappa_2} + \cdots + \frac{\Delta x_n}{\kappa_n}}{A_\times}} = -\frac{\Delta u}{R_{th}}. \quad (5)$$

Where $\Delta x_i, \kappa_i$ are the length and conductivities of the multiple layers between the heat source and end of the aluminum rod. Practically, to determine R_{th}, a parameter P_{in} is introduced, which is the amount of heat flow that is going through the rod.

B. Convection

Considering convection separately equation 1 becomes,

$$\frac{\partial u}{\partial t} = -\beta(u - u_{air}). \quad (6)$$

Equation 6 describes the change in temperature due to a differential in the rod and air temperature. A differential in rod and air temperature facilitates heat exchange. The heat exchange is governed by,

$$\frac{dQ}{dt} = h_c A_\ominus (u - u_{air}), \quad (7)$$

and one can derive equation 6 with equation 7 by relating them by equation 4. In equation 7, $A_\ominus$ is the surface area of convection. Also, the units of h_c can be seen to be, $\mathrm{Wm^{-2}K^{-1}}$. It is important to note that equation 7 is a simplification of convective heat flow. The convective coefficient, h_c has geometrical and temperature dependence, but for the purposes of this paper the temperature dependence is neglected. For situations in which the rod is vertical, compared to when the rod is horizontal, the values of h_c differ. This geometrical dependence; the fact that hot air is less dense than cold air, so the rising of hot air in the two different situations (and anything in between) are different, which effects the value of h_c.

Furthermore, one can see that equation 6 simplifies to an ordinary differential equation[2]. The solutions to equation 6 are in the form,

$$u(t) = u_{air} + (u_0 - u_{air})\mathrm{e}^{-\beta t}. \quad (8)$$

Where u_0 is the initial temperature of the aluminum rod.

C. Radiation

Lastly, isolating the effects of radiation equation 1 becomes,

$$\frac{\partial u}{\partial t} = -\gamma(u^4 - u_{air}^4). \quad (9)$$

Similar to the convection case, equation 1 reduces to an ordinary differential equation, equation 9. Furthermore, equation 9 may be solved explicitly, using integration techniques. However, the solution is not useful for the purposes of this paper, so it will be left as an exercise for the reader. Rather, the heat transfer relation is more convenient to consider,

$$\frac{dQ}{dt} = \epsilon\sigma A_\circledcirc (u^4 - u_{air}^4) \quad (10)$$

Where $A_\circledcirc$ is the surface area of radiation[3] and the unit of ϵ is 1, in other words, ϵ is dimensionless. Equation 10 describes the heat transfer via radiation between the surrounding air and the rod. Finally, one is able to derive equation 9 using equations 4 and 10.

II. EXPERIMENTAL SETUP

Five different experiments were conducted to determine the heat transport parameters. The primary materials used over the course of the experiments were:

- 0.3048 m, 22.5 mm diameter, 0.327 kg sand-blasted aluminum rod
- 15 Ω LTO100 power resistor (heat power source)
- 4 thermocouples (type-T)
- Arduino Uno Microcontroller
- DC adjustable power supply
- TMP35 room temperature sensor
- Tube insulation
- Black spray paint
- 500 ml beaker, water and ice
- Circuit breadboard and INA2126 instrumentation amplifiers
- Stand with rubberized clamps
- String
- Tgrease 2500 Series Thermal Grease

In preparation for all experiments, circuits to amplify the calibrated thermocouple readings and software code for MATLAB-Arduino integration were established. The experiments conducted are summarized in Table I.

TABLE I: Experiment number and the corresponding heat transfer parameters to be found.

Experiment number	Type of experiment	Parameters from fit
1	pure conduction	κ, c_{Al}, P_{in}
2	convection bare rod	h_c, c_{Al}, ϵ
3	coupled conduction and convection bare rod	κ, c_{Al}, h_c, ϵ, P_{in}
4	convection black rod	h_c, c_{Al}, ϵ
5	coupled conduction and convection black rod	κ, c_{Al}, h_c, ϵ, P_{in}

1. *Pure conduction*

The following experiment was designed based upon the theory discussed in Section I A and was carried out to determine the contribution of conduction.

In this experiment, 0.225 m length of insulation was wrapped around the aluminum rod to minimize the effect of convection and radiation. The rod was held vertically on a stand and the in which the clamp held the insulation. The heat source was attached to one end via nut and bolt. The operating power for the heat source was 10.42 W. Thermal paste was applied to the contact face of the heat source and the rod to increase thermal conductance. Setting the origin at the end of the rod with the heat source, four calibrated Type-T thermocouples were attached at $x = 0$, $x = 7.5$ cm, $x = 15$ cm, and $x = 22.5$ cm. Small holes were drilled in the rod, which the thermocouples were placed in. To ensure that the contact of the thermocouples with the rod was maximized, thermal paste was applied. In order to produce a secure attachment, zip-ties were used.

Furthermore, on the opposite end of the heat source, a portion of the rod was placed into a beaker filled with ice and water. Effectively, this setup kept that boundary at $\approx 0°$, since any excess heat flow from the boundary would go into melting the ice. Note that the effective length of the rod is reduced to 0.225 m since $x = 0.225$ m $\rightarrow x = 0.3048$ m of the rod was inside of the ice-water beaker ensemble.

The results of the above experiment facilitated the values of thermal conductivity, κ and the specific heat capacity of aluminum, c_{Al} and, P_{in} to be known.

Assumptions :

- Insulation made it possible to neglect convection and radiation.
- The end opposite to the heat source stayed at $0°$.
- Thermocouple calibration was accurate and precise up to experimental limitations.
- Thermocouple contact was perfect. In other words, the temperature read by the thermocouple was the temperature of the aluminum bar.
- κ was not temperature dependent.

2. *Convection with bare rod*

The design of this experiment was based upon the theory discussed in Sections I B and I C. This experiment was conducted to determine the coupling effects of convection and radiation.

A horizontal geometry was chosen for this experiment. So the convective constant, h_c corresponds to a system with cylindrical symmetry. A horizontal geometry was chosen due to the unambiguity of the surface area of convection, $A_{\ominus}$ and the surface area of radiation, $A_{\circledcirc}$. In fact, in this case, $A_{\ominus} = A_{\circledcirc}$. Additionally, if a vertical geometry were chosen, then whether the convective parameter of the system would remain constant along the length of the rod would be ambiguous. A non-constant convective parameter would add another degree of complexity to the experiment.

Two sub-experiments were carried out for convection: *Experiment A* and *Experiment B*. In Experiment A, the rod was left to heat from $\approx 0°$ to room temperature, and in Experiment B the rod was left to cool from some temperature above room temperature to room temperature. The reason these two experiments were carried out was to determine if h_c depends on the direction of convective heat flow. Note that the heat source is not attached to the bar in both of the experiments.

Experiment A: As previously mentioned, the bar was set up with horizontal geometry, in other words, the bar was parallel with the bench top. In order to conduct this experiment, first the bar was fully submerged in a box of ice, the thermocouples attached in the same manner as in Section II 1. It is imperative that the bar be surrounded by ice on all sides. If the bar is surrounded by ice, only then can it reach a uniform temperature near $\approx 0°$, which will render heat flow via conduction negligible. After enough time had passed for the bar to reach equilibrium (≈ 45 minutes) and a uniform temperature distribution, it was taken out and hung from stands with arms via strings and allowed to naturally convect and radiate.

The results of Experiment A allowed the values of the convective coefficient of the system, h_c, the specific heat capacity of aluminum, c_{Al} and emissivity of the aluminum rod, ϵ to be known.

Assumptions :

- h_c was independent of temperature.
- The aluminum bar began with a uniform temperature distribution of $\approx 0°$.
- The string had negligible contribution to the heat transport mechanisms.
- Thermocouple calibration was accurate and precise up to experimental limitations.
- Thermocouple contact was perfect. In other words, the temperature read by the thermocouple was the temperature of the aluminum bar.

Experiment B: Similar to Experiment A, the bar is to be set up with horizontal geometry. In order to conduct this experiment, the bar was submerged in boiling water. However, a beaker large enough to fit the entire rod could not be found. So to attempt to bring the bar to a uniform temperature distribution, half of the bar was put in the boiling water until it reached steady state (with the thermocouple attached), then it was flipped so the other half is in the water. When the temperatures of both halves of the rod were approximately the same, the bar was then hung from strings on stands, thus allowing the bar to convect and radiate naturally.

The results of Experiment B allowed the values of the convective coefficient of the system, h_c, the specific heat capacity of aluminum, c_{Al} and emissivity of the aluminum rod, ϵ to be known.

Assumptions :

- h_c was independent of temperature.
- The aluminum bar indeed began with a uniform temperature distribution.
- The string had negligible contribution to the heat transport mechanisms.
- Thermocouple calibration was accurate and precise up to experimental limitations.
- Thermocouple contact was perfect. In other words, the temperature read by the thermocouple was the temperature of the aluminum bar.

3. *Coupled conduction and convection with bare rod*

The following experiment was designed based upon the theory discussed in Sections I A, I B, and I C. The experiment was conducted to determine how all three fundamental modes of heat transport behave in conjunction.

The setup of the experiment was as follows: the thermocouples and heat source were placed in the same way as the experiment in Section II 1 and the bar was held horizontally (parallel to the bench top) by two stands with rubber clamps. The heat source delivered was set to deliver 5 W of power and end opposite to the heat source was simply open to the air.

The results of the above experiment allowed the values of thermal conductivity, κ, the specific heat capacity of aluminum, c_{Al}, convective coefficient of the system, h_c, emissivity of the aluminum rod, ϵ and, P_{in} to be known.

Assumptions :

- No heat transferred between the rubberized clamp and aluminum rod.
- Thermocouple calibration was accurate and precise up to experimental limitations.
- Thermocouple contact was perfect. In other words, the temperature read by the thermocouple was the temperature of the aluminum bar.

4. *Convection with black rod*

The design of this experiment was based upon the theory discussed in Sections I B and I C. This experiment was conducted to determine the coupling effects of convection and radiation. Essentially, the setup, assumptions, and goals of this experiment were identical to the experiment described in Section II 2, except only experiment B was conducted. Moreover, another difference was the rod was spray-painted black to increase its emissivity, in order to see the increase in contribution of radiative heat transfer. Also, an assumption is made that the spray-paint does not change the conductivity, κ and convective heat transfer coefficient, h_c. Lastly, another assumption was made for this experiment: the black spray paint was uniformly distributed throughout the rod.

5. *Coupled conduction and convection with black rod*

The following experiment was designed based upon the theory discussed in Sections I A, I B, and I C. The experiment was conducted to determine how all three fundamental modes of heat transport behave in conjunction with one another. The setup, assumptions, and goals of this experiment was identical to the experiment described in Section II 3. The only difference was the rod was spray-painted black to increase its emissivity, in order to see the increase in contribution of radiative heat transfer. Also, an assumption is made that the spray-paint does not change the conductivity, κ and convective heat transfer coefficient, h_c. Lastly, another assumption was made for this experiment: the black spray paint was uniformly distributed throughout the rod.

III. RESULTS AND DISCUSSION

For each individual experiment, the raw data collected was compared to a simulation that matched the experiment (see Appendix A). The simulation used a finite difference method of solving either equations 1, 2, 6 or 9, with the appropriate boundary conditions. Then, using MATLAB, a least-squared fitting technique was carried out in order to determine the parameters in which the simulation best matched the data (see Appendix B). In order to initiate the simulation, initial guesses for the heat transport parameters, c_{Al}, and P_{in} were needed. Table II summarizes the initial values inputted for the simulation.

TABLE II: Simulation parameter values.

Parameter	Value	Source
Length	0.3048 m	Measurement
Diameter	0.0225 m	Measurement
Rod segments	10	Parameter Optimization
T_{room}	20°C	TMP35 sensor
Density	2709 kgm^{-3}	Measurement
c_{al}	900 Jkg^{-1}K^{-1}	see Ref. 4
κ	205 Wm^{-1}K^{-1}	see Ref. 5
h_c	10 Wm^{-1}K^{-2}	see Ref. 6
ϵ_{bare}	0	Approximation
ϵ_{black}	1	Approximation
P_{in}	varies between experiments	Approximation

1. *Pure conduction*

Initially, it was found that the temperature of the rod was not reaching the levels predicted by the simulation. To account for the discrepancy in rod temperatures, it was found to be that all the power that the heat source outputted did go into the rod. Insulation was added to the heat source to rectify the discrepancy and bring the measured data closer to the idealized simulation.

Figure 1 displays the raw data against the least-squared fitted simulation data. It is important to note that only the thermocouples placed at $x = 0.15$ m and $x = 0.225$ m were used to produce the fitted results. These thermocouples were deemed to be properly calibrated and at a location where no externalities can effect their measurements. However, note that there still exists a discrepancy for the fitted thermocouples against the simulated data. This discrepancy is to be expected as the contact between thermocouples and rod is imperfect and the effect of insulation was not taken into consideration in the simulation. Both of these sources of error imply that the corresponding assumptions made in Section II 1 were incorrect. The remaining two thermocouples had large errors associated with them which can be easily seen on figure 1. The thermocouple located at $x = 0.00$ m measures a temperature higher than the simulation. After careful investigation, this error was deemed to be expected because that thermocouple was placed directly in contact with both the heat source and aluminum rod. This location enables the thermocouple to reach a greater equilibrium temperature than the rod. Furthermore, the thermocouple located at $x = 0.075$ m is completely in disagreement with the simulation. It was deemed that the calibration of this thermocouple was flawed.

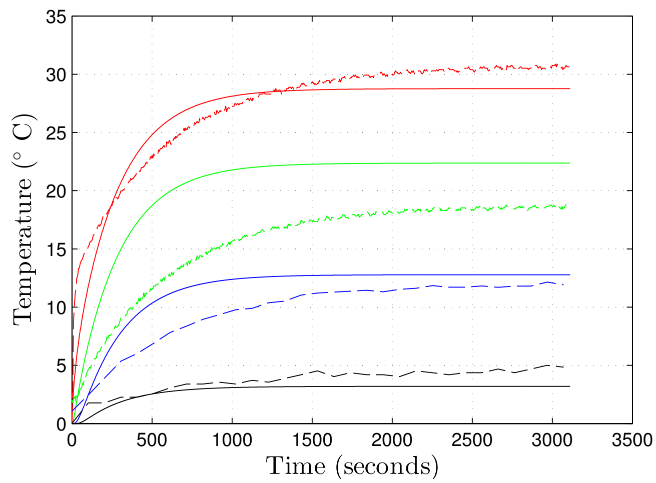

FIG. 1: Results of fitted plot to data. Solid lines represent the simulation and dashed lines represent the raw data. **Red** corresponds to $x = 0.00$ m, **green** to $x = 0.075$ m, **blue** to $x = 0.15$m and **black** to $x = 0.225$ m. Heat source is located at $x = 0.00$ m, as described in Section II 1.

The the simulated fit result in the heat transfer parameters of:

$$\boxed{\begin{array}{l} \kappa = 203.96 \text{ Wm}^{-1}\text{K}^{-1} \\ c_{Al} = 945.31 \text{ Jkg}^{-1}\text{K}^{-1} \\ P_{in} = 10.37 \text{ W} \end{array}}$$

The average error between the thermocouples and the simulated fit data is 0.1030°C, which is within reasonable experimental uncertainty. Using the P_{in} parameter determined from the fit (see figure 1), equation 5, the thermal contact resistance between the heat [7], thermal paste [8] and aluminum rod system was, $\boxed{R_{th} = 3.52 \text{ KW}^{-1}}$.

2. *Convection with bare rod*

The following section will provide results on the two experiments outlined in section II 2.

Experiment A:

As described in Section II 2 the entire bar was chilled to a uniform temperature of $\approx 5°$C and heated up to a temperature of $\approx 20°$C. Figure 2 displays the raw data against the least squared fitted simulation data. There is only one resultant fit because the theory discussed in section I B implies that there is no x dependence on convective heat transfer. From this, it can be concluded that the convective transfer coefficient, h_c is also independent of the location on the rod (for a horizontal geometry).

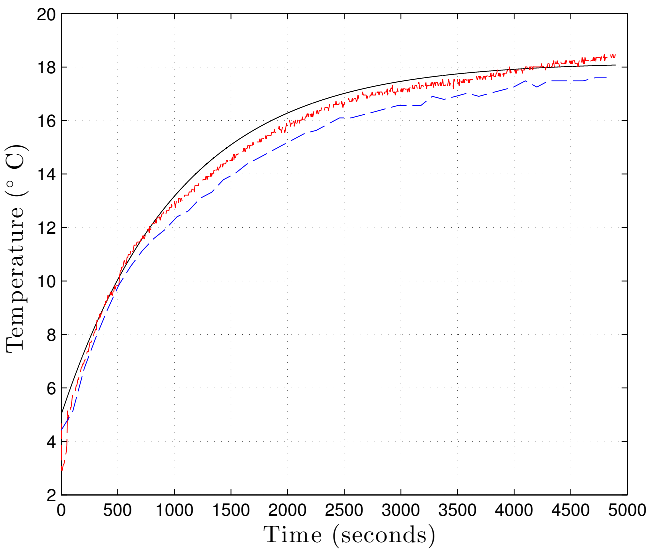

FIG. 2: Results of fitted plot to data. Solid **black** line represents the result from the fitted simulation and dashed lines represent the raw data from the thermocouples. **Red** corresponds to $x = 0.00$ m, **blue** to $x = 0.15$ m.

The the simulated fit result in the heat transfer parameters of:

$$h_c = 12.00 \text{ Wm}^{-2}\text{K}^{-1}$$
$$c_{Al} = 925.53 \text{ Jkg}^{-1}\text{K}^{-1}$$
$$\epsilon = 0.200$$

The average error between the thermocouples and the simulated fit data is 0.0228°C, which is within reasonable experimental uncertainty. Note, two thermocouples ($x = 0.075$ m and $x = 0.225$ m) were ignored as their temperature readings were unreliable due to mis-calibration. This complication does not effect the integrity of the results because it is known from Section I B that all the thermocouples should theoretically produce the same curve.

Furthermore, the thermocouples in figure 2 both record a temperature lower than the simulation. This phenomenon can be explained by the fact that the thermocouples may conduct heat from the tip of the measurement probe, therefore producing a measurement for the temperature that is lower than the temperature of the rod. This conclusion leaves the corresponding assumption made for experiment A in Section II 2 moot.

Experiment B:

As described in Section II 2 the entire bar was heated to a uniform temperature of $\approx 55°$C and heated up to a temperature of $\approx 20°$C. Figure 3 displays the raw data against the least squared fitted simulation data. There is only one resultant fit because the theory discussed in section I B implies that there is no x dependence on convective heat transfer.

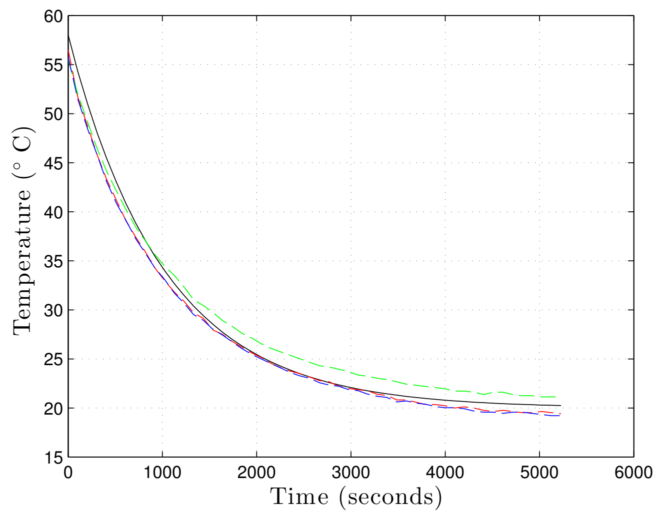

FIG. 3: Results of fitted plot to data. Solid **black** line represents the result from the fitted simulation and dashed lines represent the raw data from the thermocouples. **Red** corresponds to $x = 0.00$ m, **green** to $x = 0.075$ m, and **blue** to $x = 0.15$ m.

The the simulated fit result in the heat transfer parameters of:

$$h_c = 11.54 \text{ Wm}^{-2}\text{K}^{-1}$$
$$c_{Al} = 899.76 \text{ Jkg}^{-1}\text{K}^{-1}$$
$$\epsilon = 0.199$$

The average error between the thermocouples and the simulated fit data is 0.0214°C, which is within reasonable experimental uncertainty. Note, one thermocouple ($x = 0.225$ m) was ignored as its temperature readings were unreliable due to mis-calibration. This complication does not effect the integrity of the results because it is known from Section I B that all the thermocouples should theoretically produce the same curve.

Furthermore, the thermocouples in figure 2 show a minimal amount of discrepancy with respect to the simulation fit. This discrepancy is expected, since the method the bar was heated, explained in Section II 2, is flawed. The method may create a small temperature differential inside the bar, allowing some conduction to occur. This conduction is responsible for the slight discrepancies.

3. Coupled conduction and convection with bare rod

As described in Section II 3, all three fundamental modes of heat transport were analyzed for a bare rod. Figure 4 displays the raw data against the least squared fitted simulation data. The

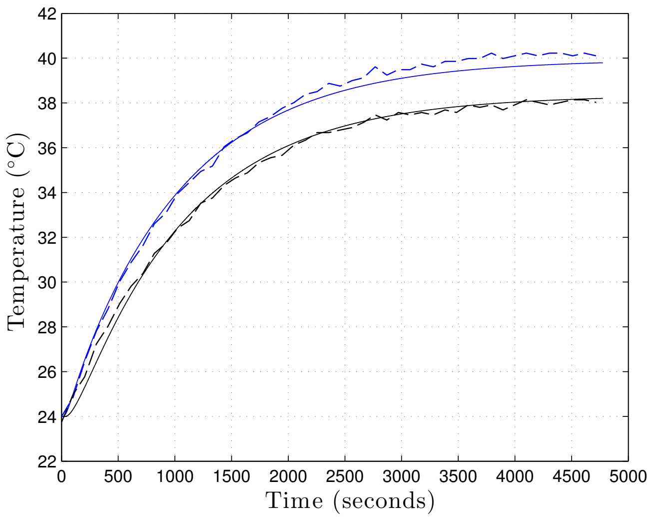

FIG. 4: Results of fitted plot to data. Solid lines represent the simulation and dashed lines represent the raw data. **Blue** to $x = 0.15$m and **black** to $x = 0.225$ m. Heat source is located at $x = 0.00$ m, as described in Section II 3.

The fitted simulation yields the physical parameters:

$$\boxed{\begin{array}{l} \kappa = 185.32\ \mathrm{Wm^{-1}K^{-1}} \\ h_c = 11.90\ \mathrm{Wm^{-2}K^{-1}} \\ \epsilon = 0.231 \\ c_{Al} = 930.02\ \mathrm{Jkg^{-1}K^{-1}} \\ P_{in} = 4.998\ \mathrm{W} \end{array}}$$

The average error from the simulation lines to the three thermocouple data sets is 0.1036°C, which is within reasonable experimental uncertainty. Two thermocouples ($x = 0.00$ m and $x = 0.075$ m) were ignored as their temperature readings were unreliable due to mis-calibration. The impact of two unreliable thermocouples was insignificant. The two reliable thermocouples were enough to generate simulated fit parameters that matched those of previous experiments. Furthermore, figure 4 shows minimal discrepancy and can be accredited to noise.

Using the P_{in} parameter determined from the fit (see figure 4), equation 5, the thermal contact resistance between the heat source [7], thermal paste [8] and aluminum rod system was, $\boxed{R_{th} = 3.14\ \mathrm{KW^{-1}}}$.

4. Convection with black rod

As described in Section II 4 the entire bar was heated to a uniform temperature of ≈ 53°C and cooled up to a temperature of ≈ 20°C. Figure 5 displays the raw data against the least squared fitted simulation data. There is only one resultant fit because the theory discussed in section I B implies that there is no x dependence on convective heat transfer.

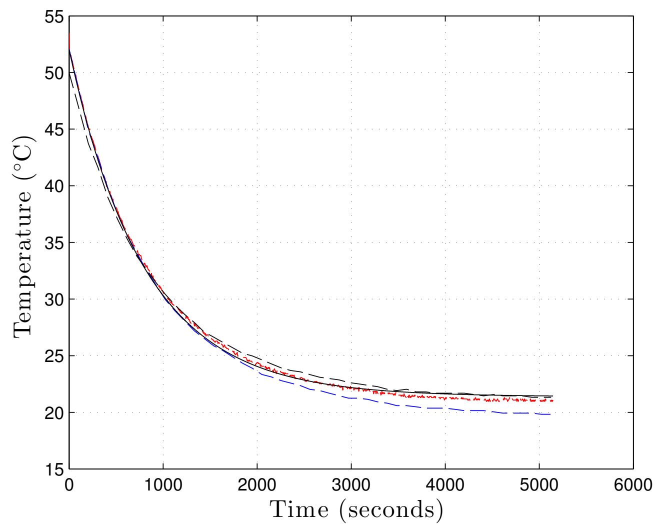

FIG. 5: Results of fitted plot to data. Solid **black** line represents the result from the fitted simulation and dashed lines represent the raw data from the thermocouples. **Red** corresponds to $x = 0.00$ m, and **blue** to $x = 0.15$ m.

The the simulated fit result in the heat transfer parameters of:

$$\boxed{\begin{array}{l} h_c = 11.94\ \mathrm{Wm^{-2}K^{-1}} \\ c_{Al} = 985.30\ \mathrm{Jkg^{-1}K^{-1}} \\ \epsilon = 0.923 \end{array}}$$

The average error between the thermocouples and the simulated fit data is 0.0268°C, which is within reasonable experimental uncertainty. Note, one thermocouple ($x = 0.075$ m) was ignored as its temperature readings were unreliable due to mis-calibration. This complication does not effect the integrity of the results because it is known from Section I B that all the thermocouples should theoretically produce the same curve.

Moreover, the thermocouples in figure 2 show a minimal amount of discrepancy with respect to the simulation fit. This discrepancy is expected, since the method the bar was heated, explained in Section II 4, is flawed. The method may create a small temperature differential inside the bar, allowing some conduction to occur. This conduction is responsible for the slight discrepancies.

5. *Coupled conduction and convection with black rod*

As described in Section II 5, all three fundamental modes of heat transport were analyzed for a black rod. Figure 6 displays the raw data against the least squared fitted simulation data.

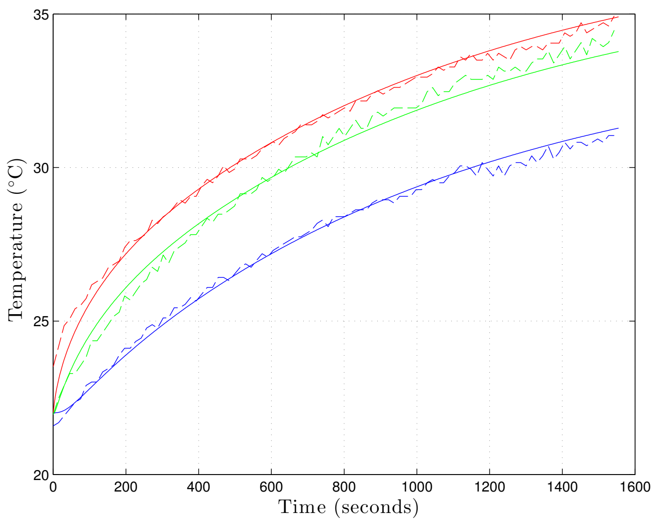

FIG. 6: Results of fitted plot to data. Solid **black** line represents the result from the fitted simulation and dashed lines represent the raw data from the thermocouples. **Red** corresponds to $x = 0.00$ m, **green** to $x = 0.075$ m, and **blue** to $x = 0.15$ m. Heat source is located at $x = 0.00$ m, as described in Section II 5.

The fitted simulation yields the physical parameters:

$$\boxed{\begin{array}{l} \kappa = 253.74 \text{ Wm}^{-1}\text{K}^{-1} \\ h_c = 12.74 \text{ Wm}^{-2}\text{K}^{-1} \\ \epsilon = 0.916 \\ c_{Al} = 985.44 \text{ Jkg}^{-1}\text{K}^{-1} \\ P_{in} = 4.05 \text{ W} \end{array}}$$

The average error from the fitted simulation to the three thermocouples is 0.0219°C, which is within reasonable experimental uncertainty. One thermocouple ($x = 0.225$ m) was ignored as its temperature readings were unreliable due to mis-calibration. The impact of the unreliable thermocouple was insignificant. The three reliable thermocouples were enough to generate simulated fit parameters that matched those of previous experiments. Furthermore, figure 6 shows minimal discrepancy and can be accredited to noise.

Using the P_{in} parameter determined from the fit (see figure 6), equation 5, the thermal contact resistance between the heat source [7], thermal paste [8] and aluminum rod system was, $\boxed{R_{th} = 3.85 \text{ KW}^{-1}}$.

Summary of Results

Compiling all the results in Section III and determining their inherent, experimental relative errors (table III) and with respect to accepted values of the heat transfer parameters, table III is established.

TABLE III: Summary of all the fitted results and their respective relative errors from the furthest empirically obtained value.

Parameter	Value
κ	$214.34 \pm 18.40\%$ Wm^{-1}K^{-1}
h_c	$12.02 \pm 6.00\%$ Wm^{-2}K^{-1}
ϵ_{bare}	$0.210 \pm 10.00\%$
ϵ_{black}	$0.867 \pm 6.46\%$
c_{Al}	$945.23 \pm 4.81\%$ Jkg^{-1}K^{-1}
R_{th}	$3.503 \pm 11\%$ KW^{-1}

TABLE IV: Summary of all the fitted results and their respective relative errors from the accepted value. See Ref. 4, 5, 6, 9 for accepted values.

Parameter	Value	Actual Value
κ	$214.34 \pm 4.56\%$ Wm^{-1}K^{-1}	205 Wm^{-1}K^{-1}
h_c	$12.02 \pm 20.20\%$ Wm^{-2}K^{-1}	10 Wm^{-2}K^{-1}
ϵ_{bare}	$0.210 \pm 10.00\%$	0.210
ϵ_{black}	$0.867 \pm 6.46\%$	0.867
c_{Al}	$945.23 \pm 5.03\%$ Jkg^{-1}K^{-1}	900 Jkg^{-1}K^{-1}
R_{th}	3.503 KW^{-1} *error not applicable*	*not applicable*

Discussion

As tables III and IV depict, there are two different types of errors that one can reason about. Most of these errors were outlined with the results in Section III, as a summary:

- Small errors in thermocouple calibrations propagating through amplifier circuits.
- No model of insulation in simulation, which leads to errors in fitting.
- Errors in unwanted heat transfer mechanisms. For example, having conduction heat transfer contributions in the pure convection and radiation experiments.
- Small errors in thermal resistance can be taken into account by the fact that uneven amounts can be applied (or the same amount is not applied each experiment).
- Thermocouples conducting heat from the rod, thus lowering the temperature measurement.
- Room temperature fluctuations may cause errors (day and night temperature fluctuations).

IV. CONCLUSION

Improvements can be made for future experimentation. Possible improvements would include increasing the accuracy of temperature measurements. This improvement would include: higher quality thermocouples and an improved method to secure the thermocouples onto the rod. A more sophisticated modeling process would improve simulation results since it will model the real situation closer than the 1-D approximation done in this paper. Additionally, carrying out the experiment in a more controlled environment would help reduce noise, temperature fluctuations and unwanted non-linear effects from the surroundings. Inside the laboratory, there can be many sources of thermodynamic heat transfer processes occurring such as: movement of humans, air conditioning, pressure, humidity, day and night temperature. If one can control as many of these as possible, a characteristic value for the heat transfer parameters can be better determined.

In closing, the heat transport parameters: thermal conductivity, κ , the convective coefficient of the system h_c, emissivity of a bare, sand-blasted aluminum rod, ϵ_{bare} and emissivity of a spray-painted black aluminum rod, ϵ_{black} were determine, as well as, the specific heat capacity c_{Al} and thermal contact resistance with an LTO100 power resistor, R_{th}. These values were obtained in a fairly accurate manner. Finally, the three fundamental modes of heat transport: conduction, convection, and radiation were discussed an analyzed for an aluminum rod.

Appendix A: MATLAB Simulation Code

The following function takes the parameters of the heat transport mechanisms and generates a simulation of temperature versus time.

```
function temperature =
    getTemperatureGradient(varargin)
% getTemperatureGradient returns the temperature
    gradient matrix of copper
% rod, with a plot of temperature vs. time at the
    specified distance x
% from the left end of the rod.
%
% The heating conditions is specified by
    heatingRodTimeStep.m, using the
% given time step dt.
%
% Each row of the temperature matrix is a snapshot
    of the rod's temperature
% at the time. The row is geographically mapped;
    i.e. the leftmost cell
% holds the temperature of the leftmost segment of
    the rod.
%
% The rod parameters can be specified with
    parameters:
%
%     parameters.rodlength  % Length of the rod
%     parameters.kappa      % Kappa value for the
    rod material
%     parameters.c          % Specific Heat Capacity
    of rod material
%     parameters.density    % Density of the rod
    material
%     parameters.crossArea  % Cross sectional area
    of rod
%
%
%   getTemperatureGradient(T, dt, fitParamName,
    guessValue, ...)
%
% returns the temperature gradient, with the
    specified parameter, given by
% fitParamName, modified to guessValue. Any number
    of fit parameters can be
% specified in this format (name, guessValue) after
    T and dt.

if isempty(varargin)
    time = 5000;
    dt = 1;
else
    time = varargin{1};
    dt = varargin{2};
end

% Initialize Parameters
parameters = struct;
timePoints = [];
timeVector = [];

setParameters();

initialConditions();

calculateTemperatureGradient;

% if length(varargin) <= 2

    plotTemperatureAt(0, 'r');
    hold on;
    plotTemperatureAt(0.075, 'g');

    plotTemperatureAt(0.15, 'b');

    plotTemperatureAt(0.225, 'k');
    xlabel('Time (s)');
    ylabel('Temperature (Celsius)');
    hold off;

% end



%% Function Definitions
    function setParameters()
        % ===== Setting parameters and stuff
        % K of Aluminum is 205 W/(m*K)
        parameters.kappa = 254.159;

        % Convection constant for Aluminum
```

```
    parameters.hConvection = 12.7353;

    % For Aluminum, at 25 Celsius, 900 J/kgC
    parameters.c = 985.47377;

    % For Aluminum, at 2700 kg/m^3
    parameters.density = 2700;

    % Diameter is 22.5 mm = 0.0225m
    parameters.radius = 0.0225/2;
    parameters.crossArea = parameters.radius^2
        * pi;

    % 1 foot
    parameters.rodLength = 0.3048;

    parameters.segments = 10;

    parameters.roomTemp = 20;

    parameters.emissivity = 0.91574;

    parameters.power = 5;

    timePoints = time/dt;

    timeVector = linspace(0, time, timePoints);

    setFitParams();

end

function initialConditions()
    temperature = ones(timePoints,
        parameters.segments) *26;
end

function calculateTemperatureGradient()
    % Skips the first row of data
    for t=2:timePoints
        lastRodState = temperature(t-1,:);

        rodState =
            heatingRodTimeStep(lastRodState, ...
            dt, parameters);

        temperature(t, :) = rodState;
    end

end

function distanceIndex = getDistanceIndex(x)
    % Calculates the index of the specified
        distance x
    if x<=0
        distanceIndex = 1;
    elseif x>=parameters.rodLength
        distanceIndex = parameters.segments;
    else
        distanceIndex =
            round((x/parameters.rodLength)*parameters.segments);
    end
end

function setFitParams()
    if length(varargin) > 2
        varNum = 3;
        while(varNum < length(varargin))

            varName = varargin{varNum};

            if ~isa(varName, 'char')
                errordlg(['Input variable '
                    num2str(varNum) ' must be a
                    string! Way to be scrub!
                    Continuing...'], ...
                   'Input Error', 'modal');

                varNum = varNum + 1;
                continue;
            end

            if ~isfield(parameters, varName)
                errordlg([varName ' is not a
                    valid parameter! Way to be
                    scrub! Continuing...'], ...
                   'Input Error', 'modal');

                varNum = varNum + 1;
                continue;
            end

            varNum = varNum + 1;
            varValue = varargin{varNum};
            if ~isa(varValue, 'double')
                errordlg([varName ' needs it''s
                    value after it! Way to be
                    scrub! Continuing...'], ...
                   'Input Error', 'modal');

                varNum = varNum + 1;
                continue;
            end
            parameters = setfield(parameters,
                varName, varValue);
            varNum = varNum + 1;
        end
    end
end

function plotTemperatureAt(distance, color)
    plot(timeVector, temperature(:,
        getDistanceIndex(distance)), color);
end


end
```

The following function uses the above simulation to generate a position versus time plot for the temperature of the rod at any position, x.

```
function temperatureAtDistance =
    getTemperatureVector( varargin )
% First input is your distance
% Next two inputs are Time and dt
% Next is a cell array of param names, followed by
    a double array of
% guesses

distanceIndex = getDistanceIndex(varargin{1});


if length(varargin) == 1
    tempMatrix = getTemperatureGradient;
else
    endTime = varargin{2};
    dt = varargin{3};
    fitParams = varargin{4};
    guess = varargin{5};
    % Making a flexible input based on variable
        number of fitParams
    input = {endTime, dt};
    for paramNum = 1:length(fitParams)
        input = [input, fitParams{paramNum}];
        input = [input, guess(paramNum)];
    end

    tempMatrix = getTemperatureGradient(input{:});
end

temperatureAtDistance = tempMatrix(:,
    distanceIndex);


end
```

Appendix B: MATLAB Fitting Code

The following code utilizes the simulation in Appendix A and performs the least squares fitting algorithm to minimize the error between the simulation plot and experimental data, extracting the fitted heat parameter values.

```
% This allows us to map correctly a simulated time
    with our actual data
endTime = ElapsedTimeseconds(end);
timePoints = length(ElapsedTimeseconds);
dt = endTime/timePoints;


%======ADJUST PARAMETERS HERE======
fitParams = {'power'};
guess = [5];
% the program automagically adjusts number of fit
    variables and stuff

errorVectorT1 = @(x) (getTemperatureVector(0,
    endTime,dt,fitParams, x) - T1);
errorVectorT2 = @(x) (getTemperatureVector(0.075,
    endTime,dt,fitParams, x) - T2);
errorVectorT3 = @(x) (getTemperatureVector(0.15,
    endTime,dt,fitParams, x) - T3);
errorVectorT4 = @(x) (getTemperatureVector(0.2250,
    endTime,dt,fitParams, x) - T4);

errorLeastSquares = @(x) sum(errorVectorT1(x).^2);




initialGuess = getTemperatureVector(0.075, endTime,
    dt, fitParams, guess);
hold on;
plot(ElapsedTimeseconds, T1, 'r');
%plot(ElapsedTimeseconds, T2, 'g');
%plot(ElapsedTimeseconds, T3, 'b');
%plot(ElapsedTimeseconds, T4, 'k');
title('Initial Guess');
hold off;

figure;



disp(['=========FIT STARTING: ' datestr(now)
    '============'])

disp('Looking for ');
for paramNum = 1:length(fitParams)
    disp([fitParams{paramNum} ', with guess '
        num2str(guess(paramNum))]);
end

disp(['Guess Error: '
    num2str(errorLeastSquares(guess))]);

start = tic;

disp('...');

[params,fval] = fminsearch(errorLeastSquares,guess);

for paramNum = 1:length(fitParams)
    disp([fitParams{paramNum} ': '
        num2str(params(paramNum))]);
end

disp(['Fit Error: ' num2str(fval)]);

disp(['Time Elapsed: ' num2str(toc(start))]);

disp(['=========FINISHED: ' datestr(now)
    '==========='']);


hold on;
plot(ElapsedTimeseconds, T1, 'r');
```

```
%plot(ElapsedTimeseconds, T2, 'g');
%plot(ElapsedTimeseconds, T3, 'b');
%plot(ElapsedTimeseconds, T4, 'k');
title('Fitted Curve');
hold off;
```

---

[1] The heat equation mentioned is the one dimensional form. The full form of the heat equation has three spacial dimensions, but is outside the scope of this paper.

[2] Convection cooling and heating is sometimes called Newton's Law of Cooling and Heating, in cases where the heat transfer coefficient is independent or relatively independent of the temperature difference between object and environment.

[3] For a horizontal cylindrical rod, $A_{\circledcirc} = A_{\ominus}$. In other words, the surface area of convection is the same as the surface are of radiation for a bare aluminum rod.

[4] "Specific heats and molar heat capacities for various substances at 20 c," http://hyperphysics.phy-astr.gsu.edu/hbase/tables/sphtt.html, accessed: 2014-07-02.

[5] "Thermal conductivity," http://hyperphysics.phy-astr.gsu.edu/hbase/tables/thrcn.html, accessed: 2014-07-01.

[6] "Heat transfer mechanisms," http://www.engr.colostate.edu/~allan/heat_trans/page4/page4.html, accessed: 2014-07-04.

[7] "Power resistor thick film technology: Lto100 power resistor," http://www.vishay.com/docs/50051/lto100.pdf, accessed: 2014-07-06.

[8] "Tgreasetm 2500 series thermal grease," lairdtech.com/WorkArea/DownloadAsset.aspx?id=1907, accessed: 2014-07-06.

[9] "Specific heats and molar heat capacities for various substances at 20 c," http://hyperphysics.phy-astr.gsu.edu/hbase/tables/sphtt.html, accessed: 2014-07-06.