

Course Name

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Assignment 1

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Question 1. Suppose V is a vector space over \mathbb{F} with a basis of order n . Then prove "Every basis of V has order n "

My Answer 1. Suppose T is another basis. Since S is independent and T is spanning, $|T| \geq |S|$. The other direction is less trivial, since T might be infinite, and Steinitz does not immediately apply. Instead, we argue as follows: since T is linearly independent, every finite subset of T is independent. Also, S is spanning. So every finite subset of T has order at most $|S|$. So $|T| \leq |S|$. So $|T| = |S|$ (copy from *Part IB - Linear Algebra*)